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# A Review of Optimal Placement of Phasor Measurement Units in Power System Distribution State Estimation

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**Abstract**

Currently, Phasor Measurement Units (PMUs) are becoming more ubiquitous in the substations, being used to collect synchronized data for many applications such as monitoring the system conditions during normal operations and protecting and controlling the power systems during abnormal situations. The OPP problem is to find out the minimum number of PMUs to make the system fully observable. Various methods have been suggested to deal with the OPP problem, including using heuristic, meta heuristics, and mathematical programing optimization strategies. Finally, this paper gives a comprehensive review of existing contributions in this wide area, concentrating on both the OPP problem and different techniques to solve it.

**Keywords**

Phasor measurement units; optimal PMU placement; optimization strategies, power system; OPP problem.

## Introduction

It is necessary for the reliable operation of power systems to observe operational conditions diligently. Control centers use data gathered in several substations to calculate based and unmetered electrical values and network parameters and identify and correct measurement and topological mistakes. Metrics were sourced from the SCADA systems in the past, such as flows of active and reactive power plus injections and voltages at the bus junctions. Phasor measurement units was developed based on the integration of the global positioning system technology and sophisticated data processing methods for relays. Sophisticated monitoring instruments that deliver accurate, time stamped plus sequence measurements are PMUs [1].
If the PMU has enough channels, the voltage phasor and all, or some, of the connected branch phasors may be directly gauged by a PMU installed on a bus. Due to the increase in deployment of PMUs in recent years, it becomes possible to perform more comprehensive monitoring, protection and control of power networks [2], [3], and [4]. In addition, although PMUs have a number of specific applications, their high unit and possibly also the cost of communication infrastructure make the choice of optimal placement of these devices questionable.

Some traditional optimization techniques are unveiled as the solution to the OPP challenge, such as combinatorial optimization, dynamic programming, linear programming and nonlinear programming. Since the traditional methods lack any ability to surmount these limitations, some advanced heuristic and modern metaheuristic optimization approaches are introduced. In this paper we discuss what are our demands, and we review the research on the most famous used heuristic and metaheuristic search methods to solve standard OPP problem. In the Section II, the problem formulation is presented, and in the third Section III, mathematical programming, in the fourth Section IV and the fifth Section V respectively, heuristics and metaheuristics are used as the solutions to the problem of the OPP. Section VI draws the conclusions.

## 2. Formulating the Optimal PMU Placement Issue

The phasor values of voltage measured at an installed bus and current measured on some or all of the connecting lines of a PMU can be measured by a PMU. These principles can be used for placement of PMUs [5]:

Principle 1: One voltage and one current measurement be used for each branch connected to a bus with a PMU.

Second, provide a voltage pseudo-measurement to each of the nodes that is seen by another bus with PMU.

Principle 3: A single current pseudo-measurement should be assigned to each branch that ties two buses with known voltages. This enables that the observed areas can be linked.

principle 4: The current is assigned to each branch with a given pseudo measurement of the current, making the current determined indirectly from Kirchhoff's current law (KCL).

This principle can be used when the current balance of a node is determined. If all other branches are known, Kirchoff's Current Law (KCL) allows calculation of the current phasor in the unknown branch. (i) The Optimal Placement Problem (OPP) is known to be extensive, (ii) and system observability is dependent on (1) the count of available PMUs and (2) a set of placements for them, and therefore, it is not straightforward to find the optimal solution. The objective function of OPP is nonconvex, non-smooth, nondifferentiable, having computational complexities manifested as nonlinearity, discontinuities, and multimodality. The following observability criteria [6], [7] must therefore be satisfied in order to determine suitable locations for PMU installations.

The PMUs at the buses have condition 1: The current phasors of all the incident branches and the bus voltage phasor are available.

Condition 2: Given voltage phasor at one end of a branch and its current phasor, voltage phasor at the other end of the branch can be derived.

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Condition 3: Given the voltage phasors at both ends of the branch, the current phasor for the branch can be directly calculated.

Condition 4, in the absence of PMU on zero injection bus, but provided current phasor of all incident branches, except one are known, KCL can be applied to ascertain the current phasor of unknown branch.

Condition 5: It is possible to use node voltage equations to find voltage phasor of a bus of zero injection type which does not have its own assigned voltage phasor, but whose voltage phasors of all surrounding buses are known.

Condition 6: Based on node voltage equations, the voltage phasors of zero injection buses can be calculated when phasor voltages of zero injection buses within a set of zero injection buses are not known, but phasor voltages of every other bus in that set are known.

The measures that were taken from Condition 1 were direct measurements. In other words, the measurements derived from Conditions 2–3 are known as pseudo-measurement. Measures derived from Conditions 4 through 6 of the multiobjective fractional programming setting are called extension-measures.

## 3. Methods of Mathematical Programming

A linear programming problem in which all decision variables are required to be integers is called an integer linear programming (ILP). Moreover, the goal of the framework described in [8] is to find the minimal number of PMUs to be strategically deployed in order to reduce measurement criticality throughout the system. We further extend the placement issue to consider standard measures as suspects of implementation. Moreover, the same framework can be used to determine the optimal locations that take into account the required degree of local redundancy. Such a way of dealing with the system facilitates building measuring devices with different sensitivity degrees which can be further employed to decrease the probability of getting wrong data and data loss.

Table 1. A Review of Optimal Placement of Phasor Measurement Units

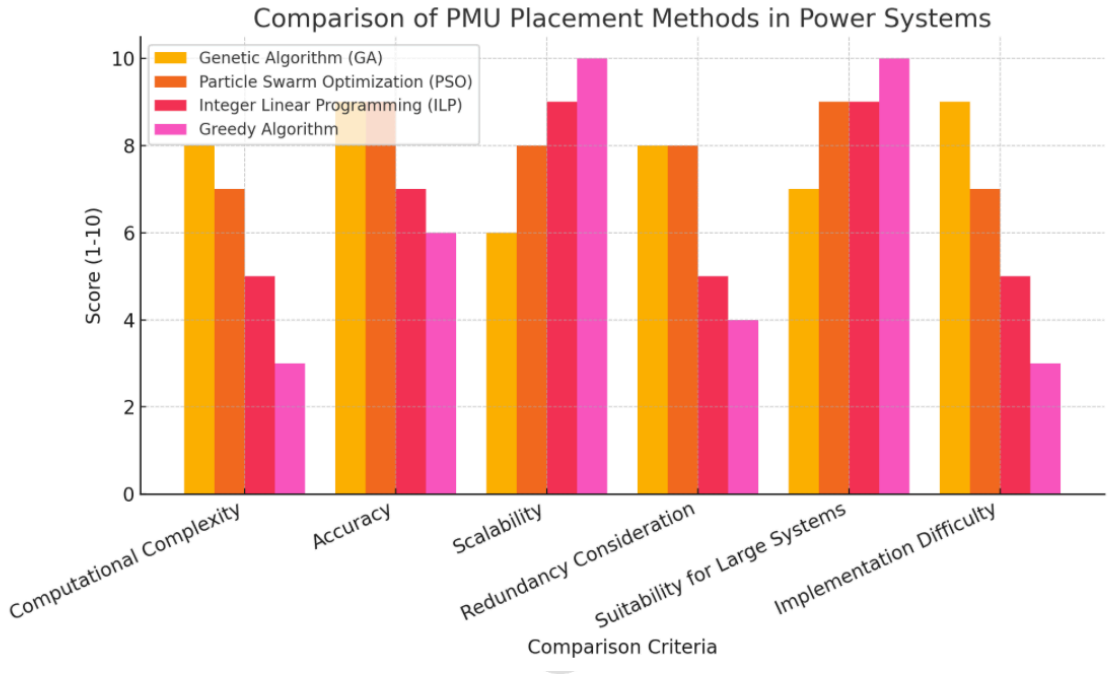
| Criteria | Method 1 | Method 2 | Method 3 | Method 4 |
|---|---|---|---|---|
| Optimization Technique | Genetic Algorithm (GA) | Particle Swarm Optimization (PSO) | Integer Linear Programming (ILP) | Greedy Algorithm |
| Objective Function | Minimize PMU count, maximize observability | Minimize cost and redundancy | Ensure full system observability with minimal units | Fast placement with acceptable accuracy |
| Computational Complexity | High | Moderate | Low | Very Low |

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| | | | | |
|---|---|---|---|---|
| Accuracy | High | High | Moderate | Low to Moderate |
| Scalability | Moderate | High | High | Very High |
| Redundancy Consideration | Yes | Yes | No | No |
| Suitability for Large Systems | Moderate | High | High | Very High |

The use of matrix reduction algorithms has resulted in reduction of pre processing, the size of the placement model, and the amount of work required to include the best possible placement set [9]. Table 1 shows the a review of optimal placement of phasor measurement units. Figure 1 shows the Comparison pf PMU placement methods in power system.

Figure 1. Comparison pf PMU placement methods in power system




## 4. Heuristic Methods

1. Depth first search (DeFS) is an algorithm for the directed graph, which partitions a graph into a forest and marks all the vertices in the order in which it was encountered and finished. In particular, the conditions of Section II are applied here. However, it is not iterative, nor is it based on the 'depth criterion'. PSAT, MATLAB — based toolkit is used to solve the OPP problem, using the DeFS approach as compared to others in [10], [11]. A further DeFS technique is suggested in [12].

2. Second, the second method uses minimum spanning tree (MST), also, a modified depth first strategy. The computing properties of the DeFSthree technique are good throughout, and it provides complex, weak convergence, but the MST technique improves convergence of the

DeFS approach. The rule adopted instead of "Find maximum bus network coverage" rule is "On ramps that link buses and off ramps that link buses". A large number of simulations in which the Yunnan Power grid in China [13], IEEE-14 as well as IEEE-30 bus systems [12] have been simulated, have been performed.

**5. Metaheuristic Methods**

1. Simulated Annealing (SA): Annealing refers to making a downward change, at each step, randomly, to the current solution; a metaheuristic approach to achieve a good solution to an optimization problem. If the cooling schedule (i.e., how fast to reduce the temperature) is slow, then the algorithm will most likely arrive at an optimal or almost optimal solution. Moreover, the SA technique implies a relatively simple objective function which takes into account the cost of the installation and the measurement of the sensors and their distribution.

2. Genetic Algorithm (GA): Genetic algorithm (GA) is a search heuristic that works on the principle of natural evolution, such as adoption of chromosomes representing probabilities. This heuristic is frequently applied to the search and optimization problems in order to generate the successful solutions to them. Using PMU placement criteria, the total number of measurements, the maximum number of measurements received from the repulsive PMU network under different condition criteria, e.g., detecting critical measurements and critical sets not included in the system; maximum estimate accuracy; minimum cost of PMU placement; etc. is computed. The transformation of the network graph

3. Tabu Search (TS) is a combinatorial search method, whereby a memory structure is used for remembering the previously visited solutions, thus allowing new combinations to be explored for optimizing the problems. In [6], the author provides a novel topological methodology by extending an augment incidence matrix and TS algorithm. With very low computation it is capable to solve the combinatorial OPP problem is very robust. Using the method for integer quantities is computationally faster and easier than standard approaches to observability analysis based on naïve analysis of matrices. Also, in [16] the TS approach on meter placement has been proposed for maximizing the topological observability.

4. Particle Swarm Optimization (PSO): PSO is a population based search methodology whose search for solutions is given by a so called particles (individuals) to a global search methodology. In a multi-dimensional search space, particles flutter around. At each time in flight each particle moves its own position in terms of its own experience and also in terms of the experience of the nearby particles but only in ther optimum place where its is and its neighbors are placed. Swarm’s direction of the particle depends on immediate neighbors of the particle and particles that have seen the particle in the past. As an optimization tool to identify the minimum number of PMUs required for complete observability, a modified discrete binary version of particle swarm algorithm (BPSO) is used from [17]. One such practice is to apply an enhanced topological observability evaluation based on topological analysis including the establishment of a new rule derived from analysis of zero injections. A BPSO algorithm is presented in [18] for the purpose of minimizing PMU installation expenses. The answer depends on few variables such as the number of branches at the proximity of it and the

communication condition on the bus. It has been demonstrated that the latter element is more suitable than the usual approaches.

**Conclusion:**

Phasor Measurement Units (PMUs) have become indispensable in today's power systems, helping to provide synchronized measurements that enhance monitoring, protection, and control. The optimal PMU placement (OPP) problem was introduced, its formulation was discussed, and the major mathematical programming, heuristic and metaheuristic approaches for achieving complete observability of the system with minimum installation cost were summarized. Mathematical programming techniques are best for smaller systems, whereas heuristic and metaheuristic algorithms are more flexible and scalable in large and more complex power networks. The future research ought to center on creating hybrid optimization methods that take into account practical parameters like communication systems, cyber-security, renewable energy incorporation, and the alteration of network conditions, to improve the dependability and efficiency of the following generation smart grid**.**

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